

Rate-Distortion Efficiency of Subband Coding with Integer Coefficient Filters¹

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We are interested in developing efficient image compression algorithms that are suitable for low-complexity implementation in deep-space probes. Subband coding or equivalent wavelet transforms provide some advantages over traditional block transforms (WHT, DCT and KLT) in terms of rate-distortion performance under the mean-square-error criterion and according to subjective visual evaluation. Furthermore, the possibility of choosing a desired resolution, independently in the time or frequency domain, is important to preserve the specific features that are most relevant for substantiating scientific findings.

A low-complexity version of a JPEG-like algorithm based on the Integer Cosine Transform (ICT), is now being implemented in software on the Galileo spacecraft. The most promising candidate for improving the current ICT-based algorithm is a subband coding method that uses quadrature mirror filters (QMF) with lattice structure and integer coefficients. Lattice QMF filters are paraunitary, perfect reconstruction (PR) filters. This provides a simple method to obtain PR filters with integer coefficients, without the need for any additional constraint to be satisfied for guaranteeing the PR property. Similarly, orthonormal wavelet transforms can be easily implemented by designing a two-channel paraunitary QMF bank and then using a tree structure. This means that the wavelet orthonormality properties can be retained even when the coefficients are quantized or subject to certain constraints.

We have used the energy compaction gain $G = \frac{(1/M) \sum_i \sigma_i}{(\prod_i \sigma_i)^{1/M}}$ as our measure of efficiency of the filters, where the σ_i are the variances in each of the M subbands, and we have considered a Gauss-Markov source with autocorrelation coefficient $\rho = 0.95$. Initially, we have computed G after rounding the coefficients of an optimal QMF lattice filter to integers in a prescribed range. Better results were obtained by using simulated annealing techniques to find optimal integer solutions. However, since high efficiency can be obtained with relatively short filters, we were able to perform an exhaustive search for optimal filters with integer coefficients in

¹The research described in this summary was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

the set $\{0, \pm 1, \dots, \pm k\}$ or with coefficients restricted to powers of two in the set $\{0, \pm 2^{\pm i}\}, i \leq k$.

Among the most promising new filters in terms of two-band gain, we found:

1. A 6-tap filter with lattice coefficients $(-4, 1, -1/4)$, where $k = 2$, having $G = 3.69$, compared to the optimal 6-tap filter with $G_{opt} = 3.79$, and to an FIR (non-lattice) filter found in [1] with $G = 3.66$.
2. An 8-tap filter with lattice coefficients $(1/4, -4, -1, -1/2)$ and $k = 2$ with $G = 3.81$, compared to the optimal 8-tap filter with $G_{opt} = 3.85$, and to an FIR (non-lattice) filter found in [1] with $G = 3.77$, which also requires a wider range for the coefficients ($k = 3$).
3. Six-tap and 8-tap integer coefficient filters with $k = 3$ and gains of 3.69 and 3.82, respectively.

Figure 1 shows the gain G achievable with lattice filters having coefficients in the set $\{0, \pm 2^{\pm i}\}, i \leq k$.

The energy compaction criterion is justified by rate-distortion theory arguments [2], and it guarantees that the distortion [3]

$$D(R) = 2\sqrt{\sigma_0^2 \gamma_0^2 \sigma_1^2 \gamma_1^2} 2^{-2R}, \quad R \geq \frac{1}{2} \log_2 \frac{\rho}{1-\rho} \quad (1)$$

is minimized if the optimal rates $R_i = R + \frac{1}{2} \log_2 \left[\frac{\sigma_i^2 \gamma_i^2}{\sqrt{\sigma_0^2 \gamma_0^2 \sigma_1^2 \gamma_1^2}} \right]$, $i = 0, 1$, are used for each subband, where R is the overall rate and γ_i are the spectral flatness measures of each subband [2]. However, we have also verified that the rate-distortion suboptimality of subband coding [3] with respect to encoding the source directly is negligible for the proposed filters, i.e. the rate-distortion performance after subband coding, computed by using equation (1), essentially coincides with that of the original source.

Compression tests have also been performed on actual images with good rate-distortion results as compared to JPEG. Other applications include the compression of seismic data, where the variable time-frequency resolution is of particular importance.

We believe that the proposed integer subband coding methods provide an improved performance/complexity solution for deep-space probes. The hierarchical property of lattice filters allows to add stages to the lattice to obtain a better filter (not necessarily optimal) without having to modify previous stages. This provides a method for adaptive decorrelation of the source. In order to further decrease



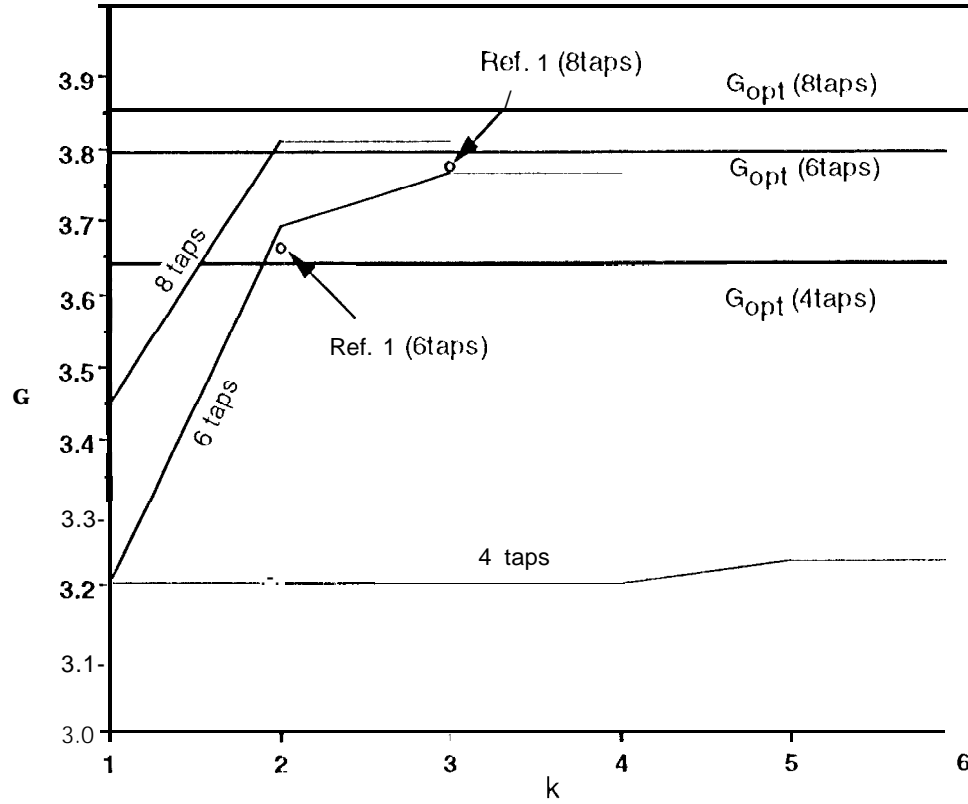


Figure 1: Gain G as a function of k for coefficients $\in \{0, \pm 2^d i\}, i \leq k$.

the complexity of subband coding methods, we are currently designing a buffering scheme that requires a total buffer of $3L \times N$ samples, where L is the number of samples in each line of the image and N is the number of taps of the filter, instead of storing the whole image.

References

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